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### Semester One Examination, 2018

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNIT 3**

## Section One:

## Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question  
have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free 35% (52 Marks)

This section has**eight (****8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

A box contains five balls numbered and . Three balls are randomly drawn from the box at the same time and the random variable is the largest of the three numbers drawn.

(a) By listing all possible outcomes (, , etc.), determine . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ lists outcomes  ✓ correct probability |

(b) Construct a table to show the probability distribution of . (2 marks)

|  |  |  |  |
| --- | --- | --- | --- |
| **Solution** | | | |
|  |  |  |  |
|  |  |  |  |
| **Specific behaviours** | | | |
| ✓ values of  ✓ values of | | | |

(c) Calculate . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates products  ✓ correct value |

Question 2 (5 marks)

A function defined by has stationary points at and .

(a) Use the second derivative to show that one of the stationary points is a local maximum and the other a local minimum. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ differentiates twice  ✓ shows and interprets  ✓ shows and interprets |

(b) Determine the coordinates of the point of inflection of the graph of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct -coordinate  ✓ correct -coordinate |

Question 3 (5 marks)

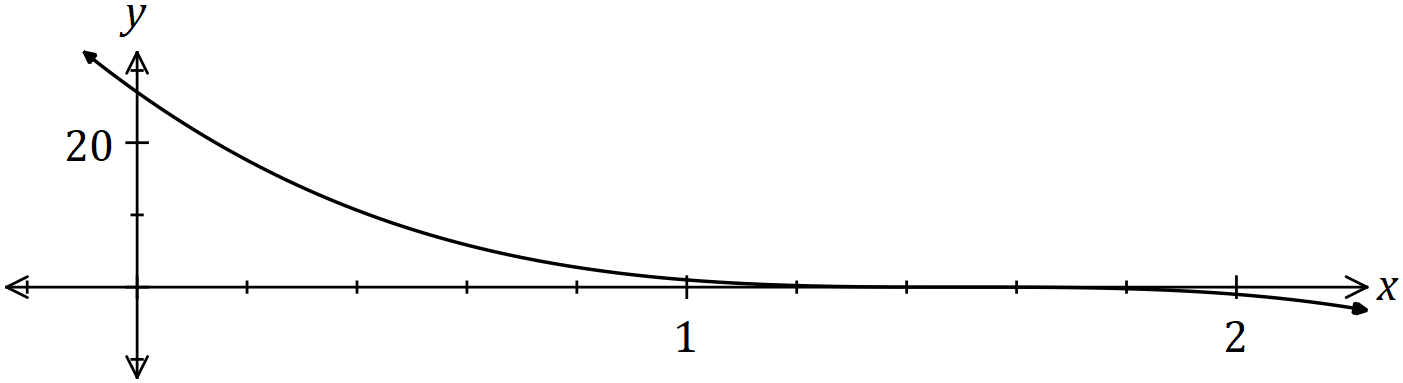
A particle travels in a straight line so that its distance cm from a fixed point on the line after seconds is given by

Calculate the acceleration of the particle when .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct form of quotient rule  ✓ simplifies expression for  ✓ correct use of chain rule in second derivative  ✓ correct expression for acceleration  ✓ substitutes and simplifies |

Question 4 (8 marks)

The graph of is shown below.



(a) Determine the area of the region enclosed by the curve and the coordinates axes.

(4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes integral with limits  ✓ antidifferentiates  ✓ expression with both limits substituted  ✓ correct area |

(b) Given that the area of the region bounded by the curve, the -axis and the line is square units, determine the value of , where . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation with antiderivative  ✓ equation with both limits substituted  ✓ simplifies equation  ✓ value of |

Question 5 (7 marks)

Determine for the following, simplifying each answer.

(a) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of chain rule  ✓ correct derivative, simplified |

(b) . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of product rule  ✓ correct derivative of  ✓ correct derivative |

(c) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ reverse limits  ✓ correct derivative |

Question 6 (5 marks)

The height, in metres, of a lift above the ground seconds after it starts moving is given by

Use the increments formula to estimate the change in height of the lift from to .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correctly uses chain rule  ✓ correct derivative  ✓ increment of time  ✓ substitutes correctly into increments formula  ✓ fully simplifies |

Question 7 (9 marks)

The function is such that , it has a point of inflection at and a stationary point at .

(a) Determine . (5 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of  ✓ value of  ✓ antiderivative  ✓ constant of integration  ✓ value |

(b) Determine

(i) . (2 marks)

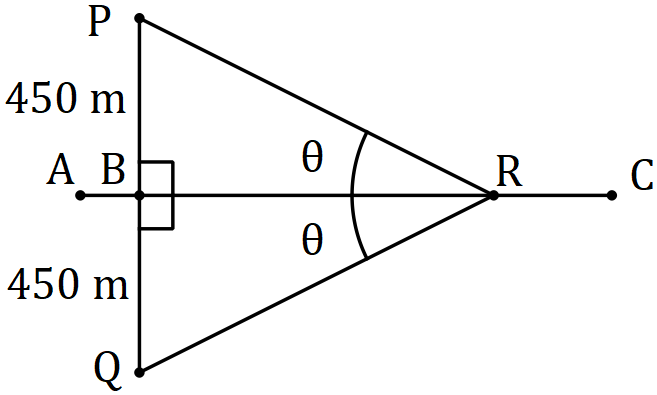
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses total change  ✓ correct value |

(ii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses linearity  ✓ correct value |

Question 8 (7 marks)

Two houses, and , are m apart on either side of a straight railway line . is the perpendicular bisector of and the midpoint of is . A small train, , leaves station and travels towards , m from .



Let , where , and , the sum of the distances of the train from the houses and station.

(a) By forming expressions for and , show that .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression for in terms of  ✓ expressions for and in terms of  ✓ expression for in terms of |

(b) Use a calculus method to determine the minimum value of . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses quotient rule  ✓ simplifies derivative  ✓ roots of derivative  ✓ minimum value of |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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